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# Ion Acoustic Turbulence, Anomalous Transport, and System Dynamics in Hall Effect Thrusters


Robert Martin[1], Jonathan Tran[2]

[1]Air Force Research Laboratory,
[2]ERC Inc.,
Edwards Air Force Base, CA USA

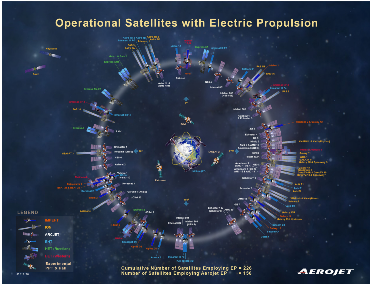


IPAM Mathematics of Turbulence Retreat, June 2017
UCLA Lake Arrowhead Conference Center, CA

# Outline

## EP-Devices:

- EP Improves Thrust/Mass (Isp)
- Ion/Elec.-Thermal/HET Flying
- FRC/MPD/Electrospray/etc. in Dev.
- Space notoriously Risk Adverse (i.e. Tech must be "Proven")




Aerojet Overview of EP Satellites (3/08)

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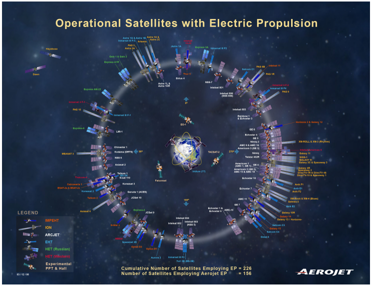
- Improved Life Estimates Needed
- Space not Replicated in Ground Test

Models needed to Bridge Gap

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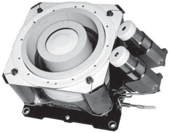
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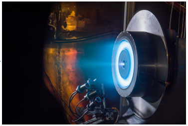

Fakel SPT-100 NASA Hermes Thruster

# Electric Propulsion

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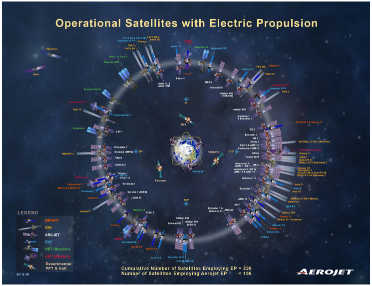


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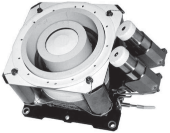

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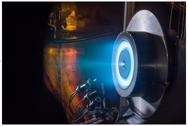

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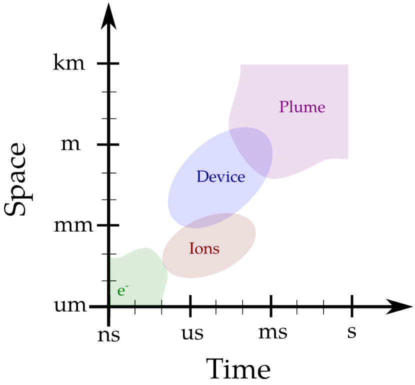
## Inherently Multiscale:

- $\mathcal{O}(10^9)$-Space and Time
- Naive 3D-Spatial Scales $\rightarrow$ Cost$^3$



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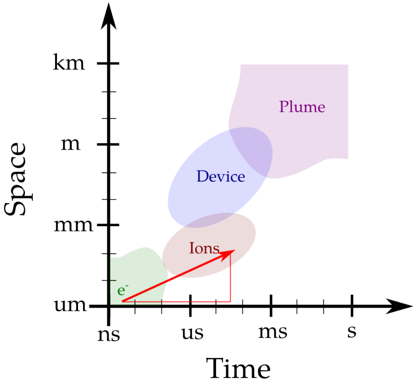


$$m_i/m_e \approx \mathcal{O}(10^{3.5})\text{-}\mathcal{O}(10^{5.5})$$

## Inherently Multiscale:

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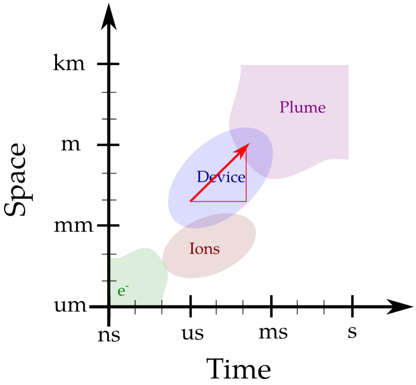

Isp=1000 Line

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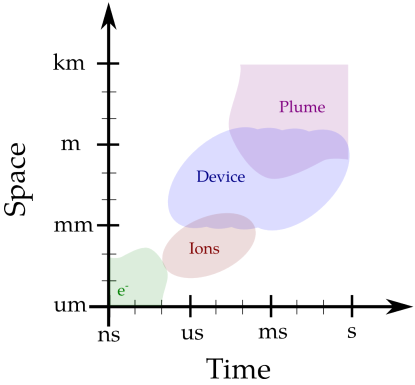


$\mathcal{O}(\texttt{GHz})$

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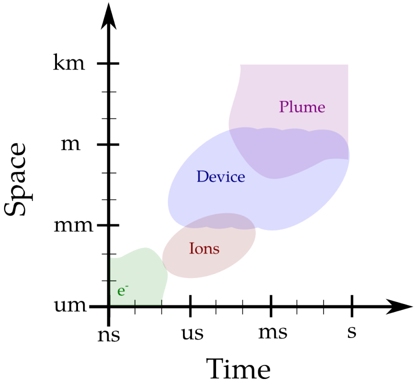


$\mathcal{O}(\texttt{GHz}) \rightarrow \mathcal{O}(\texttt{Years!})$

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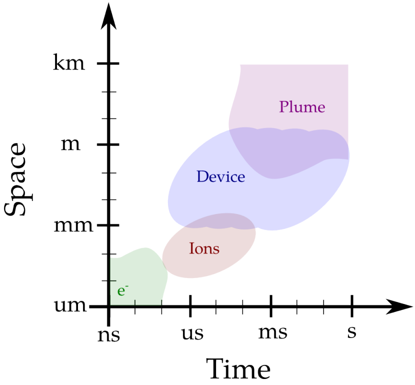
Modeling Must Exploit Scale Separation

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## Hall Effect Thrusters (HET):

- Traditionally Modeled in R-Z
- Named for Hall Current in $\theta$
- Uses Quasi-1D Electron Fluid Solve
- Ohm's Law $\rightarrow$ No $e^-$-momentum

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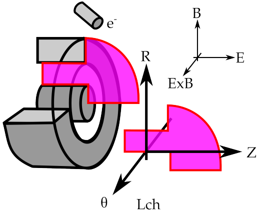

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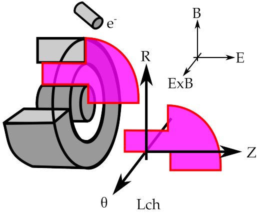


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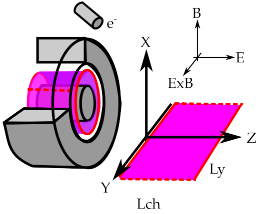

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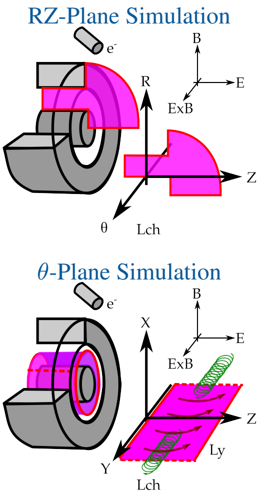

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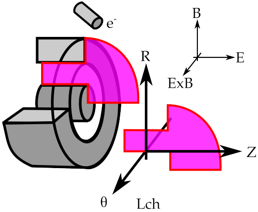


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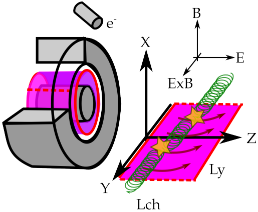

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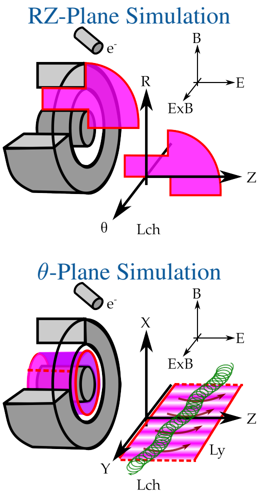

# Impact of Mobility

## Model Sensitive to Mobility:

- Classical Mobility Insufficient in Near Plume

From LaFleur, Phys Plasmas 23, 053503 (2016)

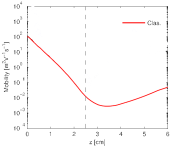


FIG. 3. Electron cross-field mobility as a function of axial position within the PPS®1350 thruster. The solid black line is an empirical mobility which is needed in fluid simulations in order to get agreement with experiment,[16] the red line is the mobility based on classical diffusion across a magnetic field, while the open blue triangles show the mobility due to the saturated instability-enhanced electron-ion friction force. The vertical dashed line indicates the thruster exit plane.

(Thruster Simulation will not “Light”)

# IMPACT OF MOBILITY

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- Enhanced Emperical “Bohm” (1/B) Mobility
- Also Needs Coefficient by “Zone” (i.e. Anode/Channel/Plume Regions)

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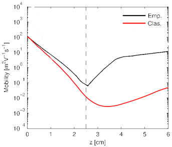


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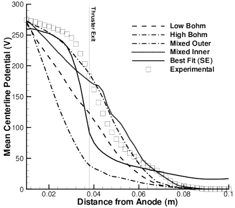
Impact of Mobility on Discharge Profile




(Koo, PhD Dissertation)

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Critical if Operating Near Mode Change

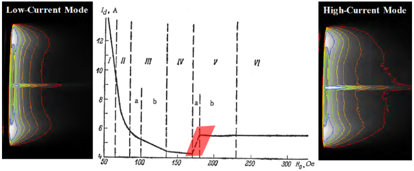


Fig. 1 Discharge current as function of magnetic field with constant discharge voltage showing operational regimes defined by Tilinin [2]

Sekerak, Gallimore, Brown, Hofer, and Polk, JPP 2016

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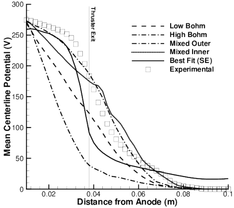


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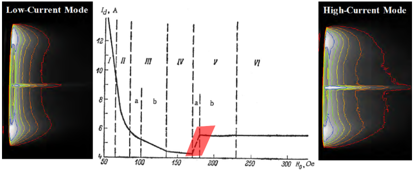
Need Physics Based Model

### Critical if Operating Near Mode Change

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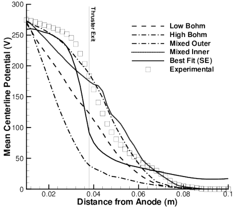


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## Current Driven Instability:

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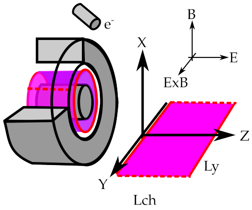
  -Anomalous e-Transport? (LaFleur)
  -Focus of 1D/2D Full-PIC Studies



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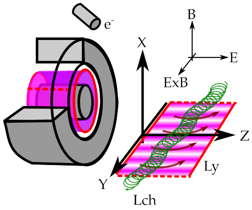


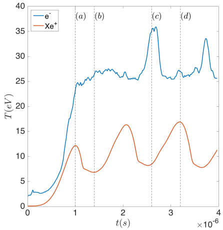

# Instability and Transport

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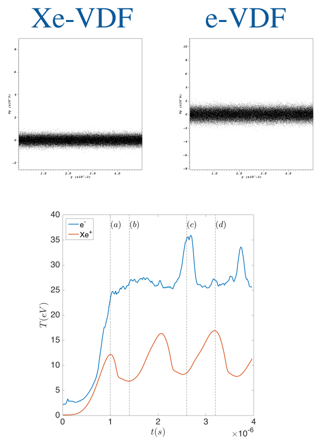

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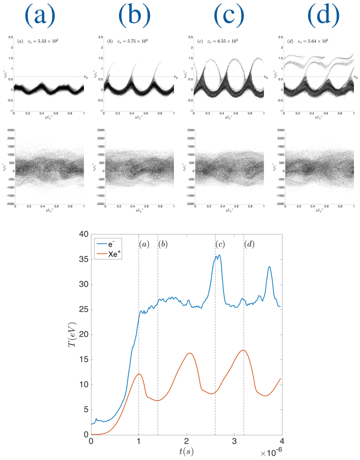

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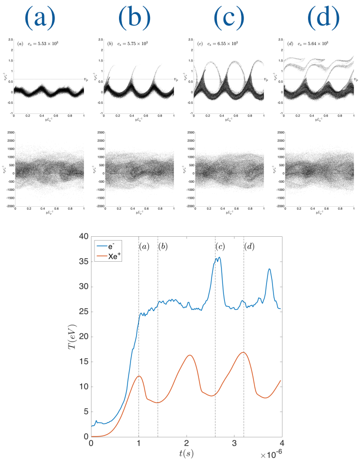

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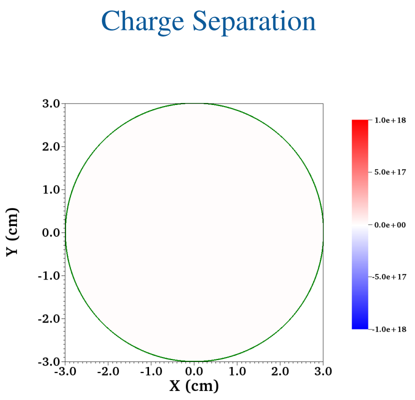
- $j_\theta \rightarrow$ FRC Spiral Charge Separation?



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- Waves not "Turbulence"?...
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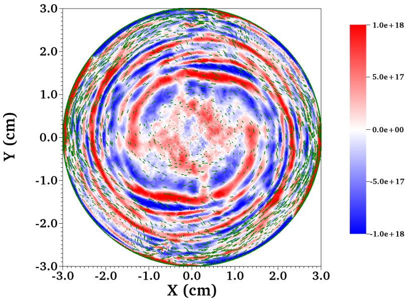
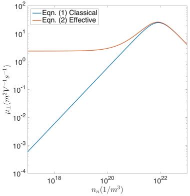
- $j_\theta \rightarrow$ FRC Spiral Charge Separation?
- Onset of Plasma Turbulence?

Charge Separation



## Impact of Ion-Acoustic Instability:

- Instability $\rightarrow$ Extra Mobility



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- Instability $\rightarrow$ Extra Mobility
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From LaFleur, Phys Plasmas 23, 053503 (2016)

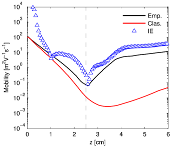
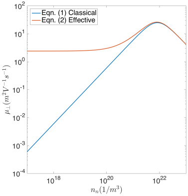



FIG. 3. Electron cross-field mobility as a function of axial position within the PPS®1350 thruster. The solid black line is an empirical mobility which is needed in fluid simulations in order to get agreement with experiment,[16] the red line is the mobility based on classical diffusion across a magnetic field, while the open blue triangles show the mobility due to the saturated instability-enhanced electron-ion friction force. The vertical dashed line indicates the thruster exit plane.

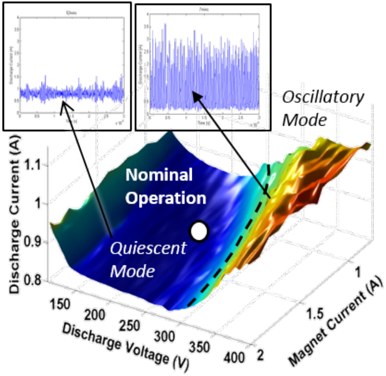
(A Posteriori Mobility)



## Impact of Ion-Acoustic Instability:

- Instability $\rightarrow$ Extra Mobility
- LaFleur Results Promising
- Need High-Dim Validation

Experimental Discharge Current Oscillations




I-V-B Plot of Hall Thruster Operation

Brown, EP TEMPEST Program Review, 2015

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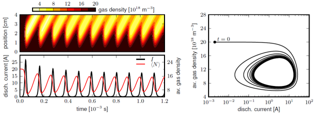


**Figure 2.** Convergence of the one-dimensional model (4) to a limit cycle with a low-current, gas-filled startup. The phase portrait shows the coupled evolution of the discharge current and of the average gas density within the domain. The parameters are $\ell = 4\,\mathrm{cm}$, $V = 250\,\mathrm{ms}^{-1}$, $Q_0 = 5 \times 10^{21}\,\mathrm{m}^{-2}\mathrm{s}^{-1}$, $\bar{I} = 4\,\mathrm{A}$, $\gamma_0 = 4 \times 10^3\,\mathrm{s}^{-1}\mathrm{A}^{-1}$, $\Psi_0 = 2 \times 10^7\,\mathrm{s}^{-1}\mathrm{m}^{-1}$. The assumed profiles $\gamma(x)$ and $\Psi(x)$ are shown on Fig. 1.

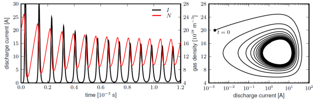


**Figure 5.** Convergence of the modified predator-prey model (9) to a limit cycle with a low-current, gas-filled startup. The parameters are derived from those of Fig. 2, assuming that the length of the ionization region represents approximately half of the discharge column length: $L = \ell/2 = 2\,\mathrm{cm}$, $Q_0 = 5 \times 10^{21}\,\mathrm{m}^{-2}\mathrm{s}^{-1}$, $\bar{I} = 4\,\mathrm{A}$, $\gamma = 3.5 \times 10^3\,\mathrm{s}^{-1}\mathrm{A}^{-1}$, $\beta \approx \int_0^\ell \Psi/\bar{N} dx = 3.5 \times 10^{-14}\,\mathrm{m}^3\mathrm{s}^{-1}$, $\tau = 2.2 \times 10^{-5}\,\mathrm{s}$.

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Barral & Peradzyński, IEPC-2009-070

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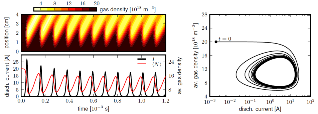


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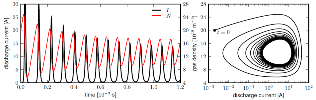


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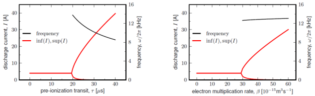


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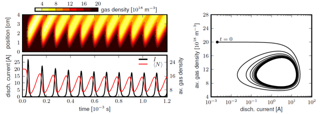
How can we Compare
Model, Experiment, and Simulation?

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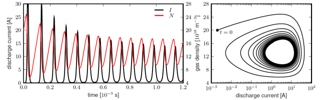


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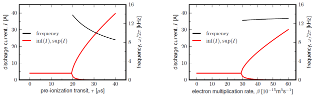


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How can we Compare
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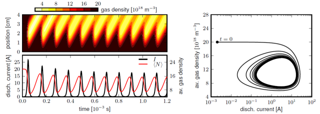
With only I(t) Accessible Experimentally?

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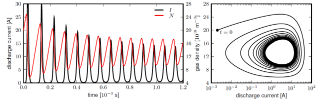


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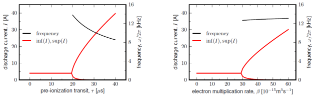


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Barral & Peradzyński, IEPC-2009-070

# Convergent Cross Mapping I.

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Same Could be Done for Hall Thruster Simulation


Same Could be Done for Hall Thruster Simulation
But Experiment has Limited Access?


State Space Reconstruction:
Takens' Theorem and Shadow Manifolds
A supplemental simulation and animation for
"Detecting Causality in Complex Ecosystems"
George Sugihara, Robert May, Hao Ye, Chih-hao Hsieh,
Ethan Deyle, Mike Fogarty, and Stephan Munch
animation by: Peter Sugihara, Hao Ye, and George Sugihara
© August 2012

# Convergent Cross Mapping II.


Shadow Maifold from only I(t)?


Shadow Maifold from only I(t)?

Useful Tuning Model/Simulation Parameters?

State Space Reconstruction:
Convergent Cross Mapping
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Useful Detecting Causal Relationships.


Useful Detecting Causal Relationships.

Potential Tool for Self-Similarity/Causality in Turbulence?

# Turbulence from Dynamic System

DMD → Tubulence:

- Low Re Vortex Shedding

Von Karman Vortex Street
http://commons.wikimedia.org/wiki/File:Karman_Vortex_Street_Off_Cylinder.ogv

## DMD → Tubulence:

- Low Re Vortex Shedding
- Sparse Dynamic System via DMD

Von Karman Vortex Street
http://commons.wikimedia.org/wiki/File:Karman_Vortex_Street_Off_Cylinder.ogv

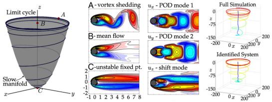


**Fig. 2.** Example of high-dimensional dynamical system from fluid dynamics. The vortex shedding past a cylinder is a prototypical example that is used for flow control, with relevance to many applications, including drag reduction behind vehicles. The vortex shedding is the result of a Hopf bifurcation. However, because the Navier–Stokes equations have quadratic nonlinearity, it is necessary to use a mean-field model with a separation of timescales, where a fast mean-field deformation is slave to the slow vortex shedding dynamics. The parabolic slow manifold is shown (*Left*), with the unstable fixed point (*C*), mean flow (*B*), and vortex shedding (*A*). A POD basis and shift mode are used to reduce the dimension of the problem (*Middle Right*). The identified dynamics closely match the true trajectory in POD coordinates, and most importantly, they capture the quadratic nonlinearity and timescales associated with the mean-field model.

Brunton, Proctor, and Kutz, 3932-3937, PNAS, 4/12/16, v.113, no. 15

## DMD → Tubulence:

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- Sparse Dynamic System via DMD
- What if Flow is Non-Sparse? (Turbulent)

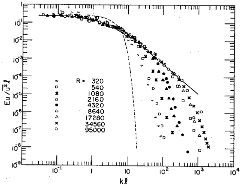
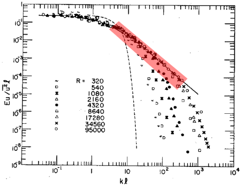



FIG. 6. Dependence of the frequency spectra $E_u/\overline{u^2}\,l$ on Reynolds number in the center of the wake.

Uberoi and Freymuth, Phys. Fluids 12, 1359 (1969).

## DMD → Tubulence:

- Low Re Vortex Shedding
- Sparse Dynamic System via DMD
- What if Flow is Non-Sparse? (Turbulent)
- Inertial Range Turbulence really Self-Similar?
- Universal if Agnostic to Large and Small $k$

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# Turbulence from Dynamic System

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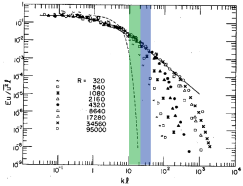
- Bandpass Invariant up to Similarity?
  (Filter Local Space/Time: Gabor? Wavelets?)

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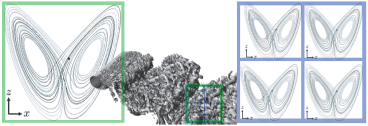

Piomelli, Phil. Trans. Royal Soc., A 372. 2014.
https://commons.wikimedia.org/wiki/File:A_Trajectory_Through_Phase_Space_in_a_Lorenz_Attractor.gif

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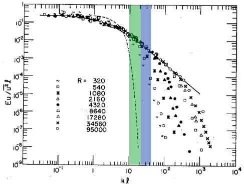
- Inter-Band Dynamics Causal via CCM?

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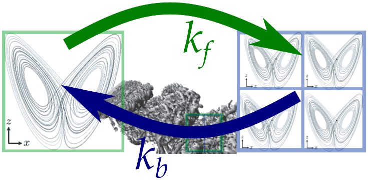


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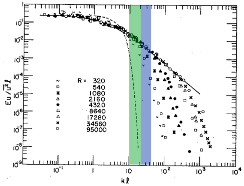
- CCM $\rightarrow K_j$ in Mori-Zwanzig LES?

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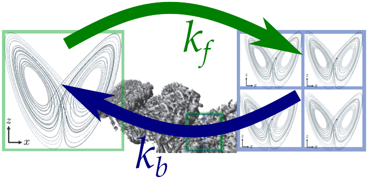


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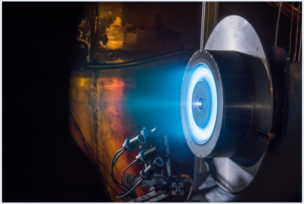
$$w_j^{(0)} = \mathcal{P} \int_o^t K_j(\overline{u}(t-s), s)ds$$

Parish and Duraisamy, AIAA Expo, 6/16

## Thank You



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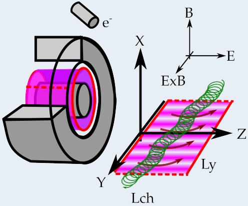


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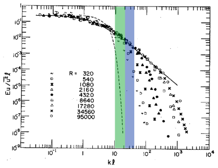


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Work Supported through AFOSR Task 17RQCOR465 (PM: Birkan)

Questions?